Language Generated by a Grammar

The set of all strings that can be derived from a grammar is said to be the language generated from that grammar. A language generated by a grammar **G** is a subset formally defined by

$$L(G)=\{W|W \in \Sigma^*, S \Rightarrow_G \mathbf{W}\}$$

If **L(G1) = L(G2)**, the Grammar **G1** is equivalent to the Grammar **G2**.

Example

If there is a grammar

G: N = {S, A, B} T = {a, b} P = {S → AB, A → a, B → b}

Here **S** produces **AB**, and we can replace **A** by **a**, and **B** by **b**. Here, the only accepted string is **ab**, i.e.,

L(G) = {ab}

Example

Suppose we have the following grammar −

G: N = {S, A, B} T = {a, b} P = {S → AB, A → aA|a, B → bB|b}

The language generated by this grammar −

$L(G) = \{ab, a^2b, ab^2, a^2b^2, \ldots\ldots\}$

$= \{a^m\ b^n \mid m \geq 1 \text{ and } n \geq 1\}$

Construction of a Grammar Generating a Language

We'll consider some languages and convert it into a grammar G which produces those languages.

Example

Problem − Suppose, L (G) = {$a^m b^n$ | m ≥ 0 and n > 0}. We have to find out the grammar **G** which produces **L(G)**.

Solution

Since L(G) = {$a^m b^n$ | m ≥ 0 and n > 0}

the set of strings accepted can be rewritten as −

L(G) = {b, ab,bb, aab, abb, …….}

Here, the start symbol has to take at least one ‘b’ preceded by any number of ‘a’ including null.

To accept the string set {b, ab, bb, aab, abb, …….}, we have taken the productions −

S → aS , S → B, B → b and B → bB

S → B → b (Accepted)

S → B → bB → bb (Accepted)

S → aS → aB → ab (Accepted)

S → aS → aaS → aaB → aab(Accepted)

S → aS → aB → abB → abb (Accepted)

Thus, we can prove every single string in L(G) is accepted by the language generated by the production set.

Hence the grammar −

G: ({S, A, B}, {a, b}, S, { S → aS | B , B → b | bB })

Example

Problem − Suppose, L (G) = {$a^m b^n$ | m > 0 and n ≥ 0}. We have to find out the grammar G which produces L(G).

Solution −

Since L(G) = {$a^m b^n$ | m > 0 and n ≥ 0}, the set of strings accepted can be rewritten as −

L(G) = {a, aa, ab, aaa, aab ,abb, …….}

Here, the start symbol has to take at least one ‘a’ followed by any number of ‘b’ including null.

To accept the string set {a, aa, ab, aaa, aab, abb, …….}, we have taken the productions −

S → aA, A → aA , A → B, B → bB ,B → λ

$S \rightarrow aA \rightarrow aB \rightarrow a\lambda \rightarrow a$ (Accepted)

$S \rightarrow aA \rightarrow aaA \rightarrow aaB \rightarrow aa\lambda \rightarrow aa$ (Accepted)

$S \rightarrow aA \rightarrow aB \rightarrow abB \rightarrow ab\lambda \rightarrow ab$ (Accepted)

$S \rightarrow aA \rightarrow aaA \rightarrow aaaA \rightarrow aaaB \rightarrow aaa\lambda \rightarrow aaa$ (Accepted)

$S \rightarrow aA \rightarrow aaA \rightarrow aaB \rightarrow aabB \rightarrow aab\lambda \rightarrow aab$ (Accepted)

$S \rightarrow aA \rightarrow aB \rightarrow abB \rightarrow abbB \rightarrow abb\lambda \rightarrow abb$ (Accepted)

Thus, we can prove every single string in L(G) is accepted by the language generated by the production set.

Hence the grammar −

G: ({S, A, B}, {a, b}, S, {$S \rightarrow aA$, $A \rightarrow aA \mid B$, $B \rightarrow \lambda \mid bB$ })